Automatic Synthesis of Random Generators for Numerically Constrained Algebraic Recursive Types*

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Abstract. In program verification, constraint-based random testing is a powerful technique which aims at generating random test cases that satisfy functional properties of a program. However, on recursive constrained data-structures (e.g., sorted lists, binary search trees, quadtrees), and, more generally, when the structures are highly constrained, generating uniformly distributed inputs is difficult. In this paper, we present Testify: a framework in which users can define algebraic data-types decorated with high-level constraints. These constraints are interpreted as membership predicates that restrict the set of inhabitants of the type. From these definitions, Testify automatically synthesises a partial specification of the program so that no function produces a value that violates the constraints (*e.g.* a binary search tree where nodes are improperly inserted). Our framework augments the original program with tests that check such properties. To achieve that, we automatically produce uniform random samplers that generate values which satisfy the constraints, and verifies the validity of the outputs of the tested functions. By generating the shape of a recursive data-structure using *Boltzmann sampling* and generating evenly distributed finite domain variable values using constraint solving, our framework guarantees size-constrained uniform sampling of test cases. We provide use-cases of our framework on several key data structures that are of practical relevance for developers. Experiments show encouraging results.

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1 Introduction

Software Testing is one of the most widespread program verification techniques, and is also one of the most practical to implement. One interesting instance of it is Property-based Testing (PBT), where programs are tested by generating random inputs and results of the output are compared against software specifications. This technique has been extensively studied, for testing correctness [20,31], exhaustiveness [34], complexity [11] etc. However, this technique requires the developer to manually write the tests, that is the properties to be checked and the random generators. The latters can be particularly complicated to design, especially in the case of complex and constrained algebraic data structures.

In this field, *constraint-based random testing* [23] (commonly used in PBT [26,13]) is a powerful technique which aims at generating random test cases that satisfy functional properties of a program under test. By specifying a property that a program has to satisfy and by using uniformly-distributed inputs generators, it is possible to uncover subtle robustness faults that may be not be discovered otherwise. For instance, [1] explored the usage of PBT for testing a steam boiler, [28] explored its usage for wireless sensor network applications. It is worth noticing that generating inputs according to a uniform probability distribution is crucial to ensure that all the distinct program behaviours have the same chance to be triggered, even those which are the most constrained. The technique has been successfully applied in the field of unit testing for imperative programs [22] as well as various programming languages including Haskell [15], Prolog [3] and proof-assistant methodologies and tools such as Coq [32] or Isabelle [10]. Sampling constraint systems solutions according to a uniform distribution is a well-known difficult problem. Initially studied in the context of hardware testing [30], the problem has been studied in [21] and more extensively in [2]. Other random generation schemes are either not uniform, or very slow *e.g.* rejection sampling is generally uniform by construction, but fits very poorly with generation under constraints.

Recently, in [38] the authors introduced an automated framework capable of providing tests for functions that manipulate constrained values without requiring manual input from the programmer. The framework introduces a type language, with algebraic data-types, and constrained types *i.e.* types augmented with a membership predicate that is used to filter invalid representations. To verify that a function does not create invalid representations, the authors opted for a random testing approach. The main interest of the framework is that both the generators and the specifications are automatically extracted from the constraints specified by the user, which greatly alleviate the user's workload. Generators are uniform random value samplers used to provide input for functions, and specifications that are predicates that verify that a given value satisfies the constraint attached to its type, are used to check whether a function's output violates the constraint or not. Their tool is implemented as a pre-processor for OCaml programs, *i.e.* before compiling, programs are rewritten into augmented programs where a test suite has been added. During the pre-processing step, from each constrained type declaration τ is extracted a CSP p. Then, each p is

solved only once, that is to say that a characterisation of the set of solutions of p, called coverage, is calculated. Each coverage is then compiled into code which uniformly generates solutions which are then converted back into values of the type τ. However, to be able to solve a CSP only once per constrained type, the authors limit themselves to types involving a fixed number of numerical atoms (*e.g.* tuples), which automatically excludes recursive types. This makes it impractical as, for instance, in OCaml, real-world programs rely heavily on recursive data-types (lists, trees, sets, etc.).

This paper investigates the automatic synthesis of uniform pseudo-random generators, as in [38], but for recursive constrained types.

1.1 Contributions

- A programmable method to restrict the values a recursive type can take.
- An algorithm that uses Boltzmann generation and constraint solving to automatically derive uniform generators for recursive constrained types.
- An experimental study of the performances of our technique.

1.2 Outline

This paper is organised as follows: Sec.2 presents use cases of our methods on some examples of realistic code. Sec.3 defines our solving technique which mixes Boltzmann generation and global constraint solving. Sec.4 recalls some elementary notions about Boltzmann sampling and details some specifics about our use-case. Sec.5 presents our prototype and gives some details about its functioning, current capabilities and restrictions. We also give some details about our implementation and measure experimentally the performances of the generators we derive for recursive constrained types. Sec.6 describes some related work. Finally, Sec.7 summarises our work and discusses possible future improvements.

2 A Declarative Programming Approach

We propose a testing framework that allows programmers to specify constraints on recursive data structures. From these constraints, the framework extracts a Constraint Satisfaction Problem (CSP) which is solved in such a way that uniform random instances (i.e., test cases) are generated. These instances are then used for testing functions in order to find defects.

2.1 Preliminaries

A pseudo-random generator g for an algebraic data-type τ is a function g of type $\mathcal{S} \to \tau$. Here, $\mathcal{S}$ is the random state used by the pseudo random number generator. A constrained type is a pair $\langle \tau, p \rangle$, with τ an algebraic data-type and $p : \tau \to \mathit{bool}$ a predicate over values of type τ. The set of its inhabitants is

defined as $\{t \in \tau \mid p(t) = true\}$. A pseudo-random generator g for a constrained type $\langle \tau, p \rangle$ is a function $g : \mathcal{S} \to \tau$ s.t $\forall s \in \mathcal{S}, p(g(s)) = true$.

Here, we face two main challenges for automating random testing of recursive data-types. First, we have to equip the developer with convenient means for specifying constraints attached to a given data-type. For example, we want to express that a list of integers is sorted or that a tree is a binary search tree (i.e., the left child node value is always smaller than the right one). Second, building an uniform random value generator for constrained recursive data-types is highly challenging. Recursive types can dynamically grow to an arbitrarily large size and, deriving generators for such types requires the resolution of a complex constraint system. In particular, we have to manage CSPs with an a-priori unknown number of variables and constraints. The grammar of Ocaml types and constraints annotations are given in Figure 7. In the following, we give two illustrative examples.

2.2 Example 1: Inserting an element into a set of integers

Let start with `list`, a recursive data-type associated to lists of integers, for which a possible type declaration is given in Fig.1. Using `list` to specify a

```
type list = Empty | Cons of int * list
```

Fig. 1: OCaml type declaration of lists of integers

Set data structure can easily be done using Testify, by using the annotation `[@@satisfying _]` and the `[alldiff]` constraint as illustrated by Fig. 2.

```
type uniquelist =
  | Empty
  | Cons of int * uniquelist [@@satisfying alldiff]
```

Fig. 2: OCaml type declaration of sets of integers using lists

We can automatically test the functions that manipulate instances of the `uniquelist` type by checking if they break the properties attached to it. For that, we have to define a generator and a specification for the corresponding type. To randomly generate instances, we first draw at random an instance of size n using Boltzmann generation (see Sec.4), then we build a CSP $(\mathcal{X}, \mathcal{D}, \mathcal{C})$ containing n finite domain variables and solve it using Path-oriented Random Testing (PRT) (see Sec.3) and eventually we build a random generator g able to produce uniformly distributed sets of size n.

A key aspect of Testify is based on the usage of *global constraints*, which are arithmetic-logic constraints holding over a non-fixed number of variables. In the example of Fig.2, we translate the declaration `[alldiff]` into a `all_different` global constraint implementation and used it to generate uniformly distributed solutions that can be used to polulate test cases. For other recursive data-types, we use combination of multiple global constraints and arithmetic

constraints. Possible recursive data-types that can be implemented and tested in our framework include functions that generate and manipulate (un-)ordered lists and sets, trees, binary search trees, quadtrees, etc.

Fig.3 shows an example of a function which implements the insertion of an element within a set of integers and the code that is automatically generated for the testing of this function[6].

```
let rec add (x:int) (l:uniquelist) : uniquelist =
  match l with
  | Empty -> Cons(x,Empty)
  | Cons(h,tl) -> if x <> h then Cons(h,(add x tl) else l)

(* generated code*)
let add_test () =
  let size = Random.int () in let rand_x = Random.int () in
  let rand_l = unique_list size in
  assert (alldiff_checker (add rand_x rand_l))
```

Fig. 3: Insertion of an element into a set, and the generated corresponding test

Here, testing the function function means verifying that every output produced is indeed sorted (`assert (alldiff_checker (add rand_x rand_l))`. Note that we have used the return type annotation to automatically derive a (partial) specification for the function, but the generator we automatically synthesise can also be used to test any hand-written specification.

2.3 Example 2: Binary Search trees

Binary Search Trees (BST) are binary trees that additionally satisfy the following constraint: the key in each node is greater than or equal to any key stored in the left sub-tree, and less than or equal to any key stored in the right sub-tree. Stated differently, the keys in the tree must be in increasing order in a depth-first search traversal, in infix order. From this observation, we propose, using our framework, a possible OCaml declaration for BSTs illustrated in Fig.4.

```
type bst =
  | Node of bst * (int[@collect]) * bst
  | Leaf [@@satisfying fun x -> increasing x)]
```

Fig. 4: Testify type annotation for binary search trees

Rather than defining global constraint for all user-declared data-types, we break the problem in two parts. On the one hand, we define or reuse known global constraints for lists, and on the other hand we define a way to browse

[6] The predicate `alldiff_checker` checks that the result list does not contain duplicates. It should not be mixed with the version of `alldiff` used in the type declaration which is used to generate randomly distributed solutions of that constraint

data structures, in a certain order, by collecting the components that are subject to a global constraint. This is done using the `(int[@collect])` annotation.

Also, the order in which the structure is explored is crucial as it determines the order in which the variables will be passed to the global constraint. By default, a depth first order is assumed. For constructors with several arguments (*e.g.* `Node`), and for tuples, the order in which the traversal is made is mapped on the declaration order of the tuple component, that is in traversal order. Fig.5 shows the code generated that traverses the tree.

```
let rec collect = function
| Node (a, b, c) ->
    List.flatten [collect a; Collect.int b; collect c]
| Leaf -> []
```

Fig. 5: Generated collector for binary trees

Here, the primitive `Collect.int` is a primitive of our framework that takes an integer and builds the singleton list with this element. This way, we first visit the left sub-tree, the root and the right sub-tree. Using pre-order or post-order would give different results. This means that the constructor `Node` must be declared in the above order and, for example, the following would be invalid: `Node of (int[@collect]) * binary_tree * binary_tree`. However, this restriction can easily be lifted by providing an annotation which would allow the programmer to explicitly specify the traversal order. Similarly, a global annotation `[@bfs]` (resp. `[@dfs]`) could be used to specify that the structure must be traversed using a breadth first search (resp. depth first search). This will be studied in future work.

3 Constrained Type Solving

A *Constraint Satisfaction Problem* (CSP) is a triple $(\mathcal{X}, \mathcal{D}, \mathcal{C})$ where $\mathcal{X}$ is a set of variables, $\mathcal{D}$ is a function associating a finite domain (considered here as a subset of $\mathcal{Z}$ without any loss of generality) to every variable and $\mathcal{C}$ is a set of constraints, each of them being $< var(c), rel(c) >$, where $var(c)$ is a tuple of variables $(X_{i_1}, .., X_{i_r})$ called the scope of c, and $rel(c)$ is a relation between these variables, i.e., $rel(c) \subseteq \prod_{k=1}^{r} D(X_{i_k})$. For each constraint c, the tuples of $rel(c)$ indicate the allowed combinations of value assignments for the variables in $var(c)$. Given a CSP $(\mathcal{X}, \mathcal{D}, \mathcal{C})$, an *assignment* is a vector $(d_1, .., d_n)$, which associates to each variable $X_i \in \mathcal{X}$ a corresponding domain value $d_i \in D(X_i)$. An assignment satisfies a constraint c if the projection of $\mathcal{X}$ onto $var(c)$ is a member of $rel(c)$. The set of all satisfying assignments is called the solution set, noted $sol(\mathcal{C})$. A constraint c is said to be *satisfiable* if it contains at least one satisfying assignment, it is inconsistent otherwise. A CSP $(\mathcal{X}, \mathcal{D}, \mathcal{C})$ is satisfiable if it contains at least one assignment which satisfies all its constraints (i.e., $sol(\mathcal{C}) \neq \emptyset$). A global constraint is an extension of CSPs with relations concerning a non-fixed number of variables. For instance, the *sort(Xs, Ys)* global constraint

[29] takes as inputs two lists of n finite domain variables Xs, Ys (where n is unknown) and states that for each satisfying assignment $(d_1, .., d_n, d'_1, .., d'_n)$ of the constraint, we have $\forall j, \exists i$ s.t. $d'_j = \sigma(d_i)$ and $d'_1 \leq .. \leq d'_n$, where σ is a permutation of $[1..n]$. Filtering a global constraint $c(X_1, .., X_n)$ with the *bound-consistency* local filtering property means to find D' such that for all i, the extrema values of $D'(X_i)$ are parts of satisfying assignments of c.

3.1 Path-Oriented Random Testing

Path-oriented Random Testing (PRT) is basically a divide-and-conquer algorithm, introduced in [22], which aims to generate a uniformly distributed subset of solutions of a CSP. Starting from an initial filtering step result, the general idea is to fairly divide the resulting search space into boxes of equal volumes and, after having discarded inconsistent boxes using constraint refutation, to draw at random satisfying assignments.

More precisely, applying constraint filtering results in domains that can be over-approximated by a larger box (i.e., an hyper-cuboid) that contains all the filtered domains. Based on an external division parameter k, PRT then fairly divides the box into k subdomains of *equal* volume. When a subdomain cannot be divided according to the division parameter k, then it is simply extended until its area can be divided. The iteration of the process leads to a fair partition of the search space into k^n subdomains where n is the number of variables of the CSP. Then constraint refutation can be used to discard (some) subdomains which are inconsistent with the rest of the CSP. As all subdomains have the same volume, it becomes possible to sample first the remaining subdomains and then, second, to randomly draw values from these subdomains. Note that, when all the subdomains are shown to be inconsistent, then the CSP is shown to be inconsistent. This contrasts with reject-based methods which will trigger assignment candidates and will reject them afterwards, without terminating in a reasonable amount of time.

Input: CSP:$(\mathcal{X}, \mathcal{D}, \mathcal{C}), k, N$: #Sol. - **Output:** $t_1, .., t_N$ or $\emptyset$ (Inconsistent)
$\mathcal{D}' :=$ `boxfilter`$_{\texttt{bc}}(\mathcal{X}, \mathcal{D}, \mathcal{C})$; $(H_1, .., H_p) :=$ `Fairly_Divide`$(\mathcal{D}', k)$; $T := \emptyset$;
while $N > 0$ *and* $p \neq 0$ **do**
  Pick up uniformly H at random from $H_1, .., H_p$;
  if H *is inconsistent w.r.t.* $\mathcal{C}$ **then**
    remove H from $H_1, .., H_p$
  else
    Pick up uniformly t at random from H and remove it;
    if $\mathcal{C}$ *is satisfied by* t **then**
      add t to T ; $N := N - 1$;
    end
  end
end
return T;

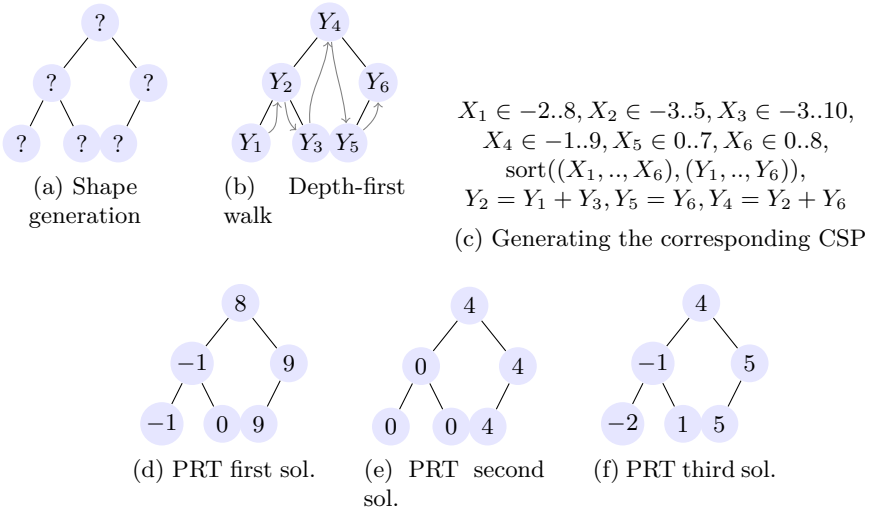
Algorithm 1: Path-Oriented Random Testing adapted from [22] to the uniform random generation of N solutions of a CSP

The PRT algorithm, adapted from [22] to the case of CSP solution sampling, is given in Figure 1. It takes as inputs a CSP, a division parameter k, and N a non-negative integer. Here, we make the hypothesis that, if the CSP is consistent, it contains more than N solutions. The algorithm outputs a sequence of N uniformly distributed random assignments which satisfy the CSP. If the CSP is unsatisfiable, then PRT returns $\emptyset$. After an initial filtering step using bound-consistency, the algorithm partitions the resulting surrounding box in subdomains of equal volume (`Fairly_Divide` function). Then, for each locally consistent subdomain H in the partition, value assignments are randomly selected and checked against the constraints of the CSP. Those which do not satisfy the constraints are simply rejected. As shown in [22], this process ensures the uniform generation of tuples in the solution space.

3.2 Extension with Global Constraints

Handling global constraints is a natural extension of PRT as it allows us to handle recursive constrained data-types. As the shape of the data structure is unknown at constraint generation time, the number of variables to be handled is also unknown in the general case. Thus, using global constraints in this context is particularly useful as it allows us to avoid the decomposition of a global constraint into the conjunction of several simpler constraints. This results in both a stronger and faster pruning. In order to handle recursive constrained data-types, we had to provide a dedicated interface for accessing the deductions from global constraint solving. To facilitate the access to global constraints, we created an API which provides results of PRT over different global constraint combinations. The API provides access to predicates such as `increasing_list(+int LEN,+int GRAIN,-var L)` in which `L` is instantiated to a list of `LEN` uniformly distributed random integers ranked in increasing order, and the random generator is initialised with `GRAIN`. Optionally, the predicate can be called with domain constraints in order to constrain the returned list of values in specific subdomains. Other similar predicates are provided as part of the API, namely `increasing_strict_list/3 (+int LEN,+int GRAIN,-var L)` which returns a list of strictly increasing integers; `decreasing_list/3` (resp. `decreasing_strict_list/3`) which provides a list of integers in (resp. strict) decreasing order or else `alldiff_list/3` which returns a list of uniformly distributed random distinct integers. PRT can also be used in combination with any available global constraint and arithmetico-logic constraint. The following example, given in Fig.6 illustrates how PRT is used in this respect.

In this example, PRT is used with one global constraint, namely $sort(Xs, Ys)$, and some domain and arithmetic constraints to populate a constrained binary search tree (BST) of size 6. In this example, the shape of the BST is unknown and some constraints hold over the keys: the domain of the key-variables is constrained (from an externally specified source), *e.g.*, key $X_1 \in -2..8$, key $X_2 \in -3..5$, etc. and any key of the BST corresponds to the sum of its children (if any), *e.g.*, $Y_{father} = Y_{child_l} + Y_{child_r}$. Note that the keys have to be set in increasing order to correspond to a valid BST. Note also that we ignore in



(a) Shape generation (b) Depth-first walk (c) Generating the corresponding CSP

(d) PRT first sol. (e) PRT second sol. (f) PRT third sol.

Fig. 6: Generation of a constrained BST of size 6. Division parameter=2, length of seq.=3, 60 subdomains over 64 have been discarded after the first filtering.

which order will the keys be positioned in the tree. The first step of our method corresponds to the generation of a uniformly distributed random shape of the BST (Fig.6(a)) using the Boltzmann method, described in Sec.4. Then, a depth-first walk along the tree assigns variable identifiers to the nodes and collects the constraints that must hold over the constrained data structure (Fig.6(b)). The generated CSP (Fig.6(c)) can then be solved by using PRT, which generates, in this example, three uniformly distributed random solutions (Fig.6(d)(e)(f)). It is worth noticing that other uniform random solutions sampling methods such as [21,36] could have been used in this context. PRT was chosen because of its availability and simplicity. However, non-uniform random sampling such as a simple heuristic selecting at random variable and values to be enumerated first would not have been appropriate in this context as the goal was to test the robustness of user-defined functions in functional programming.

4 Boltzmann Sampling

The Boltzmann method was introduced in [17] as an algorithmic method to derive efficient sampler from *combinatorial classes*. Combinatorial classes are just sets of discrete structures with a size (a non-negative integer) and such that the number of structures having the same size is finite. For example, the binary trees whose size is the number of leafs is a combinatorial class, but binary trees whose size is the length of leftmost branch is not because the number of binary trees with a leftmost branch of fixed length k is infinite. We briefly present the method here and refers the reader to [17] for more details.

In the context of that paper (similarly to [12]), that method directly translates into an automatic way to derive a uniform random generator of terms for

⟨*decl*⟩ ::= 'type' ⟨*type identifier*⟩ '=' ⟨*type*⟩ — type declaration
{'[@@satisfying' ⟨*constraints*⟩ ']'} — Testify's annotation

⟨*type*⟩ ::= ⟨*coretype*⟩
| ⟨*sumtype*⟩

⟨*coretype*⟩ ::= 'int' | 'float' | 'char' | ... — basic types
| ⟨*coretype*⟩ {'*' ⟨*coretype*⟩ } — product
| ⟨*type identifier*⟩

⟨*sumtype*⟩ ::= ⟨*constructor identifier*⟩ {'[@collect]'} 'of' ⟨*coretype*⟩
| ⟨*sumtype*⟩ {'|' ⟨*sumtype*⟩ }

⟨*constraints*⟩ ::= 'alldiff' | 'increasing' | 'decreasing' — SICStus global constraints
| ⟨*arith*⟩ — arithmetic constraints like in [38, Fig. 1]
| ⟨*constraints*⟩ {'&&' ⟨*constraints*⟩}

Fig. 7: Syntax of OCaml algebraic data-types (ADT) with Testify's annotation

the type language whose syntax is given in Fig.7. In our case, the produced generators only generate a *shape* of tree structure in a first step and the content of this shape is provided in a second step by a constraint solver which makes sure to fill the shape with values that satisfy the specified constraints. For each constrained recursive type declaration, we must therefore generate a glue function between the shapes generated by the Boltzmann sampling method and the solutions returned by the solver used. This function is illustrated in the case of binary search trees in Fig.8

```
let rec fill_binary_tree shape solutions =
  match shape with
  | Label ("Node", [x1; x2; x3]) ->
      let x1 = fill_binary_tree x1 solutions in
      let x2 = Testify_runtime.to_int x2 solutions in
      let x3 = fill_binary_tree x3 solutions in
      Node (x1, x2, x3)
  | Label ("Leaf", []) -> Leaf)
```

Fig. 8: Generated function for filling the shapes for binary search trees.

Here, we consider types as sets of terms (the inhabitants of the type) whose size is the number of `[@collect]` values they contain. For example, using `type binary_tree` of Sec.2, the term `Node(Node(Leaf, 3, Leaf), 25, Leaf)` has size 2.

In the following, we denote $\Gamma\mathcal{A}_x$ a Boltzmann sampler of parameter x for the set $\mathcal{A}$. Such sampler produces an object $\gamma \in \mathcal{A}$ with a probability $\frac{x^{|\gamma|}}{A(x)}$ where $|\gamma|$ is the size of γ and $A(x)$ is a normalizing factor called *generating series*[7]. Note that objects of the same size have the same probability to be drawn.

[7] The generating series $A(z)$ of a combinatorial class $\mathcal{A}$ is defined by $A(z) = \sum_{\gamma \in \mathcal{A}} z^{|\gamma|}$

The second interest of Boltzmann samplers is that they compose well with sum, product and substitution *i.e.* the constructors of ADTs. Fig.9 shows the derivation of such samplers. At the end of the generating process, the object

```
type t = a * b (* a and b are 2 types previously defined *)
let gen_t x = gen_a x, gen_b x

type u = A of a | B of b
let gen_u x =
  if random() < A(x)/(A(x) + B(x))
  then gen_a x else gen_b x

type alst = Nil | Cons of a * alst (* alst(z) = 1 + z · alst(z) *)

let rec gen_alst x : alst =
  if random() < 1/(1 − A(x))
  then Nil else Cons(gen_a x, gen_aList A(x))
```

Fig. 9: Sampler derivation using Boltzmann

drawn has a random size, but we see in the previous code that the choice of the parameter x influences the size. Note that we can precisely and efficiently compute x to target a size (see [6] or [33] for the details).

Still, the size is random. The last ingredient is to choose a parameter ϵ (which does not depend of the targeted size n) and keep only objects of size between $n - \epsilon$ and $n + epsilon$. Thus, the size of the object is kept up to date during the generation and the generation is stopped if that size exceeds the upper bound $n+\epsilon$. At the end, the object may be smaller than $n-\epsilon$ in which case it is rejected too. However, the theory (see [17]) guarantees that the rejections cost remain relatively low, *i.e.* the cumulated size of objects sampled to obtain an object of size in the interval $[n - \epsilon, n + \epsilon]$ is in $\mathcal{O}(n)$. So the complexity of the overall process is linear in the size of the generated object.

An important point to mention is the case of polymorphic types. From a theoretical point of view they fit in the framework. But from a practical point of view it is hard to sample a "polymorphic value". To deal with that limitation, the Boltzmann samplers are instantiated only for concrete types *e.g.* not for `'a list` but for `int list`.

5 Implementation and Experiments

We have implemented the work presented in the previous sections in a tool available at the url `https://github.com/ghilesZ/Testify`. Our implementation relies on several state-of-the-art tools. The derivation of OCaml code from annotated OCaml source files is done using the *ppx* framework, as in [37,5], which is a form of generic programming [24]. Pre-processors using *ppx* are applied to source files before passing them on to the compiler. They can be seen as self-maps over abstract syntax trees. In our case, the source files are traversed to

find OCaml type declarations and derive their associated generators. These generators are then used to provide inputs for the functions that must be tested. We have implemented the techniques presented here for the global constraints that we have been able to identify in real data structures (BSTs, Sets, etc) namely *alldiff*, *increasing* and *decreasing* (both strict and large versions). Note that to extend our implementation, *i.e.* add a global constraint,, it is sufficient to add to the constraint solver a propagator for the said global constraint, as both the step of traversing the structure and the random generation procedure presented in section 3 being common to all types.

The work done by Testify is divided into two phases: the first is the pre-processing phase during which our tool collects some information on the types needed to build the generators. The second is the testing phase, where the generated code is executed to produce inputs for the functions under test. Note that the pre-processing phase is performed only once while the testing phase can be triggered multiple times, each time one needs to run the tests.

We distinguish four kinds of types, for which we provide four different synthesis techniques:

- For non recursive unconstrained types (*e.g.* `int`, `float * (int * int)` ...) we determine at pre-processing time the function to be used as a generator. For that, we rely on the qcheck [15] library, which provides the primitives for building and composing generators.
- For non recursive constrained types (*e.g.* `int[@satisfying fun x -> x >=0]`), we extract a single CSP which is solved once, still at pre-processing time. From this resolution is extracted a code that draws uniformly solutions of this CSP and rebuild from them a value of the corresponding type. This is the method described in [38].
- For recursive unconstrained types (*e.g.* lists, binary trees), we build samplers by using the Arbogen [19] tool. This tools implements the Boltzmann method presented in Sec.4. The tuning of the Boltzmann parameter is done at pre-processing time while the shape generation, and the conversion of this shape to a value of the targeted type is done at testing time.
- Finally, for recursive constrained types (*e.g.* sorted lists, binary search trees), the previous techniques are mixed together to produce efficient generators: first, a targeted size n is drawn, then, a shape of size n is sampled. We then browse the generated shape, collecting constrained values to build a CSP as explained in Sec.3. This CSP is then fed to the SICStus Prolog [4] solver, which builds from it a generator using the *PRT* library [22]. Finally we put together shapes and constrained values. All of these steps are made at testing time, that is every time we have to generate a value we must solve a CSP. This is arguably the bottleneck of our architecture, but experiments still demonstrate the usability of our method.

5.1 Experiments

In this section, we focus on the performance of our automatically derived generators. We measure the generation times (in seconds) obtained with our method

for different constrained recursive types and by varying the size of the generated structure. The types we are interested in are: lists sorted in ascending order, association lists with unique keys, lists of pairs in ascending order ($(x, y) \leq (x', y) \Leftrightarrow x \leq x' \wedge y \leq y'$), binary search trees (unbalanced), functional maps (key-value stores as binary search trees) and quadtrees. These types are among the most frequents in the literature, and they only involve numerical constraints, which Testify is able to manage.

Types	**Targeted**	**Average**	**♯Objects**	**time**
increasing_list	10	8.50	2889	0.020
	100	93.95	13691	0.004
	1000	948.57	17763	0.003
	10000	9392.45	79	0.757
assoc_list	10	8.49	2534	0.023
	100	93.96	11949	0.005
	1000	947.87	13660	0.004
	10000	9406.92	76	0.786
bicollect	10	6.99	2492	0.024
	100	93.04	6418	0.009
	1000	947.73	16048	0.003
	10000	9456.85	1596	0.037
binary_tree	10	9.00	238690	0.001
	100	94.35	21214	0.001
	1000	948.00	3416	0.006
	10000	9740.00	1500	0.040
map	10	9.00	238690	0.001
	100	94.37	21208	0.001
	1000	947.08	3423	0.006
	10000	9047.00	1276	0.047
quad_tree	10	8.00	3590507	0.001
	100	93.88	228357	0.001
	1000	947.79	23548	0.002
	10000	9489.19	2191	0.027

Fig. 10: Generation time per object according to the size of the structure

The experience was to sample as much as possible constrained structures during one minute. The results are shown in Fig.10. For each type we report the size of the terms (number of `[@collect]` values) targeted, the average size of the generated terms, the number of terms sampled and the average time to sample one term. The computer running the experiments has an Intel Core i7-6700 CPU cadenced at 3.40GHz with 8 GB of RAM.

As expected, at least for the tree-like types, we observe that the complexity is quite linear in the size of the sampled terms: the Boltzmann method keeps its promises and the use of a constraint solver proves to be fast enough to be used in our context. For most of these structures we manage to generate several hundred values per second, up to a certain structure size. These results prove the relevance of our method in the context of testing, as it can allow the user to fine-

tune the generators to decide whether he wants to test his functions on several small structures and/or a few large ones. However, we may note that sampling of lists is much slower than sampling of trees. This is due to the fact that the Boltzmann method is not tailored for regular languages (such as list). It would probably be more efficient to use specialised algorithms for regular languages such as the one of [8].

6 Related Work

In this section we focus on related work dealing with constraint-based generation techniques. Constraint-based generation of test data has been exploited in white-box testing to produce inputs that will follow some execution paths, as well as in functional testing to generate constrained inputs. In [35], Senni applies constraint logic programming to systematically develop generators of structurally complex test data, e.g. red-black trees, in the context of Bounded-Exhaustive Testing.

PBT, as exemplified by Quickcheck for Haskell, has been adapted to many programming languages but also to proof assistants to test conjectures before proving them, e.g. [18,10,32,13]. In [18] restricted classes of indexed families of types are provided with surjective generators. In [13], the authors propose the FocalTest framework for testing - conditional - conjectures about functional programs and for automatically generating constrained values. In this work, CP global constraints are not used and thus FocalTest does not take benefit from the corresponding efficient filtering ad hoc procedures.

In the context of PBT of Erlang programs, De Angelis *et al* propose in [16] an approach to automatically derive generators of values that satisfy a given specification. Generation is performed via symbolic execution of the specification using constraint logic programming. A difference between their approach and ours is that we craft a suitable representation of a given type at static time, which is then compiled into an efficient generator. In [16], generators are built at execution time, while testing, which ultimately leads to a slower generation.

The Coq plugin QuickChick helps to test Coq conjectures as soon as involved properties are executable. It allows the automatic synthesis of random generators for algebraic data-types, recursive or not, and also the definition of simple inductive properties, e.g. a property specifying binary search trees whose elements are between two bounds, to be turned into random generators of constrained values [25]. The approach is narrowing-based, like in [14]. Such a binary tree is built lazily while solving the constraints found in the inductive property while in Testify, the shape of the data structure is randomly chosen and then its elements are obtained by solving constraints. This tool comes with different primitives or mechanisms allowing for some flexibility in the distribution of the sampled values. For example the user can annotate the constructors of an inductive data-type with weights that are used when automatically deriving generators. Furthermore, it also produces proofs of the generators correctness.

In [12], the authors adapt a Boltzmann model for random generation of OCaml algebraic data-types, possibly recursive, but not constrained. Generators are au-

tomatically derived from type declarations. In [14], Claessen et al. propose an algorithm that, from a data-type definition, a constraint defined as a Boolean function and a test data size, produces random constrained values with a uniform distribution. However the authors show that this uniformity has a high cost. They combine this perfect generator with a more efficient one based on backtracking. Limiting the class of constraints and combining it with an efficient solving process, Testify can generate constrained values with a uniform distribution in a reasonable time. Some work focus on the enumeration or sampling of combinatorial structures, like lambda-terms, using Boltzmann samplers [27], Prolog mechanisms [9] or both [7]. These approaches are dedicated to objects of recursive algebraic data-types with complex constraints, like typed lambda-terms, closed lambda-terms, linear lambda-terms, etc. This kind of constraints is out of reach of our tool whose objective is not only to generate constrained values but also to provide the programmer with syntactic facilities to specify them.

7 Conclusion

We have proposed in this paper a technique based on declarative programming, to derive generators of random and uniform values for constrained recursive types. We have proposed a small description language for recursive structure traversal which allows us to build a custom CSP for each term to be generated. The code we generate is efficient, and outperforms a naive generation technique based on rejection, and allows us to generate large recursive structures quickly. Starting from the constraints attached to a type, we first sample the shape of the value to generate and then build a CSP that encodes the valid representations of the terms that have this shape. Then, our tool uses the SICStus Prolog constraint solver to filter invalid representations and produce a uniform solution sampler. Our technique is integrated into the Testify framework, which embeds these generators within a fully automatic test system. The generators derived by our framework are fast enough to allow the user to run tests each time he compiles his code. This would allow him to be able to detect bugs very quickly and fix them before they become potentially harmful. However, we still have a lot of work to do to improve Testify. For example, we can extend the constraint language to be able to handle types with shape constraints (*e.g.* balanced trees). This would require adapting the Boltzmann technique to random sampling of tree structures under constraints. Also, when dealing with a functional language, functions as values cannot be avoided: it will be necessary to have techniques for the derivation of generators for functions, and explore what kind of constrained functions (monotonic, bijective functions, etc.) appear in practice in programs. Moreover, in this paper we have only studied *tree-like* recursive data-structures. Some structures do not fit into this framework (*e.g.* graphs, doubly linked lists) and it would be interesting to see to what extent our methods adapt to these structures. Also, our current implementation tests functions by generating any random input, disregarding their body. This is naturally an important point of

improvement. For example, one could imagine a static analysis of the body of the function, to conduct the input generation more precisely, and find bugs faster. Finally, our framework targets OCaml but the methods developed in this paper can be adapted to most programming languages and proof assistants.

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